

MATH 112A Review: Integration by parts

Facts to Know:

Formula for integration by parts:

$$\int_a^b u dv = u \cdot v \Big|_a^b - \int_a^b v du$$

Easy way to remember:

$$\begin{array}{cc} u = \text{ } & dv = \text{ } \\ du = \text{ } & v = \text{ } \end{array}$$

Examples:

1. Find $\int x \sin x dx$. =I $u = x$ $dv = \sin x dx$
 $du = 1 \cdot dx$ $v = -\cos x$

$$\begin{aligned} I &= x \cdot (-\cos x) - \int (-\cos x) dx \\ &= -x \cos x + \int \cos x dx \\ &= -x \cos x + \sin x + C \end{aligned}$$

Check our work:

$$\frac{d}{dx} (-x \cos x + \sin x + C) = -1 \cdot \cos x + (x)(-\sin x) + \cos x = x \sin x.$$

2. Find $\int \ln x dx$. =I $u = \ln x$ $dv = dx$
 $du = \frac{1}{x} dx$ $v = x$

$$\begin{aligned} I &= x \ln x - \int \frac{1}{x} \cdot x dx \\ &= x \ln x - x + C. \end{aligned}$$

3. Find $\int e^x \sin x \, dx = I$

$$\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array} \quad \begin{array}{l} dv = e^x \, dx \\ v = e^x \end{array}$$

$$I = e^x \sin x - \left[\int e^x \cos x \, dx \right]$$

$$\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array} \quad \begin{array}{l} dv = e^x \, dx \\ v = e^x \end{array}$$

$$I = e^x \sin x - \left[e^x \cos x + \int e^x (-\sin x) \, dx \right]$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$I = e^x \sin x - e^x \cos x - I$$

$$\Rightarrow 2I = e^x \sin x - e^x \cos x$$

$$I = \frac{e^x \sin x - e^x \cos x}{2} + C.$$